Quantum Chaos, Transport, and Decoherence in Atom Optics

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Abstract

Introduction: Classical and Quantum Chaos

Quantum mechanics has been a rich field of study for so long because it provides many phenomena that run counter to our classically rooted intuitions. While such counterintuitive behavior occurs in simple systems, such as transmission through a double-slit aperture or an entangled pair of particles, even more surprising behavior arises in the study of quantum systems whose classical counterparts are chaotic.

The study of chaos dates back to Poincaré's analysis of the stability of planetary motion near the end of the 19th century. Since then, it has been found to be a ubiquitous phenomenon in many areas, from the study of plasma confinement and weather to disease epidemiology. For our purposes, we can define chaos to be deterministic yet unpredictable behavior in a simple system. The unpredictability in chaotic systems is commonly referred to as "sensitive dependence on initial conditions," which we can restate more formally as follows. Suppose the state of a system is parameterized by a set of variables $x_1, \ldots, x_n$ (which determine the *phase space* for the system). The evolution of these variables with time determines a *trajectory* for the system. Then chaotic systems are characterized by exponentially rapid separation in time of nearby pairs of trajectories. For predictability, this property means that a necessarily imperfect measurement of the initial condition or rounding errors in a numerical calculation will make long-term prediction of chaotic trajectories impossible.

One natural question that arises is how the complicated behavior associated with chaos becomes manifest in quantum systems. Clearly, the notion of chaos as it is usually defined is classical, because quantum mechanics is a theory of waves, not trajectories. In fact, as we will see in the discussion of dynamical localization below, quantum effects *suppress* classical chaos. Thus quantum systems with sensible classical counterparts never show chaotic behavior. This statement appears to pose problems for the Correspondence Principle, if we are to believe that one can recover classical behavior as a limit of quantum theory. Nonetheless, the quantum dynamics of classically chaotic systems are intimately connected with the underlying classical dynamics. This dissertation studies two different examples of the transition from quantum to classical mechanics and the "fingerprints" of classical chaos in quantum mechanics.

Atom Optics

The research described here is primarily experimental, and the setting for the experiments is atom optics. The field of atom optics is generally concerned with the manipulation of atoms using electromagnetic fields or material objects. In the experiments described here, very cold (hundreds of nK to tens of μK) cesium atoms are manipulated by forces due to laser light.

Specifically, we use an *optical lattice*, which is formed by two (linearly polarized) laser beams propagating in opposite directions. These two beams interfere to form a stationary, sinusoidal intensity pattern in space. The laser is tuned close enough to an atomic resonance to gain a resonant enhancement of the laser-atom coupling, but the tuning is far enough away that spontaneous scattering of the laser light is suppressed. In this regime, the behavior of an atom in the optical lattice is particularly simple: the atom feels an energy shift proportional to the local intensity (the *ac Stark effect*), and to a very good approximation we can ignore the internal atomic dynamics and dissipative effects on the atomic motion. Thus the atom behaves as a point particle in a sinusoidal potential, which is just a realization of the quantum pendulum. Of course, this technique is very general and powerful: any intensity pattern that can be produced (e.g., using holographic techniques) will literally act as a spatial potential for atoms. For the experiments here we work exclusively with the simple sinusoidal pattern in space, but we induce classical chaos by modulating the intensity of the laser light in time.

Performing experiments where atoms interact with light naturally requires a clean source of atoms. The setup for the experiments here uses a standard magneto-optic trap (MOT). A MOT consists of six appropriately polarized laser beams and the magnetic field from a pair of coils. The net result is a cloud of cold, confined cesium atoms inside a vacuum chamber, providing a convenient initial condition for optical-lattice experiments. To perform the experiments in the optical lattice, the trapping fields are extinguished so that they do not interfere with the atomic dynamics in the lattice.

This setup is ideally suited to the study of quantum chaos for a number of reasons. First, the atoms are well isolated, leading to long decoherence times. This aspect is particularly important when studying coherent quantum effects. Second, standard optical tools provide a high degree of precise control over the optical lattice, and a number of atom-optical techniques allow for good control over the initial state of the atoms. Third, the fact that the atoms are cooled and prepared in a trap allows for relatively long (ms) interaction times with the optical potential. Finally, there is a straightforward method for measuring the momentum distribution of the atoms. In this *ballistic expansion measurement*, the atoms are released from the optical lattice and are allowed to drift freely in the dark for some amount of time. They are then frozen in place by the MOT laser light, and the spatial distribution of atoms is photographed by a CCD camera. Because the distance the atoms move during the drift time is proportional to their momentum, the CCD image is a direct measurement of the momentum distribution. Of course, this measurement technique is destructive, and each measurement requires that the whole experiment be repeated.

Dynamical Localization and Decoherence in the Kicked Rotor

The first set of experiments in this dissertation is concerned with transport in the kicked-rotor problem, a textbook system in the study of quantum and classical chaos. We realize this system experimentally by simply turning on the optical lattice in a series of short, periodic pulses. Intuitively, one might expect the atoms to gain kinetic energy as they are "kicked" by the laser light. More precisely, because of the classical chaos, they undergo *diffusive* motion in momentum. This transport, which is equivalent to a random-walk process in momentum space, is characterized by a mean kinetic energy that grows linearly with time (and a momentum distribution whose width grows as $\sqrt{t}$). For our purposes, we can regard this diffusive transport as an indicator of chaotic behavior.

When we examine the quantum dynamics of the kicked rotor, things are quite different, as we can see from the simulations in Fig. 1. The quantum simulation shows diffusive behavior only for short times, after which the diffusion rate drops off to nearly zero. Because the quantum momentum distribution stops diffusing and "freezes in" to a nearly stationary profile, this effect is termed *dynamical localization*. Roughly speaking, this effect is is a result of quantum interference, where the transport to larger momenta is suppressed due to destructive interference. This effect is one manifestation of the quantum-mechanical suppression of chaotic behavior that we noted above.

This effect poses obvious problems for quantum–classical correspondence. The absence of chaotic behavior in the quantum dynamics makes it difficult to see how classical behavior can be viewed as a limiting case of the quantum behavior. Since the quantum and classical diffusion curves agree for short times, one might expect that this time scale should simply become very long in the classical limit. While it does grow longer as one moves a system towards the classical limit, it has been argued that the rate at which it grows is much too slow (scaling only logarithmically with the action scale of the system) to explain our everyday classical world. Thus if one were to walk into a machine shop and build a macroscopic version of the kicked rotor, these scaling arguments would predict absurdly short (ms) time scales where quantum effects should become important.

One approach to solving this correspondence problem is embodied in the theory of *decoherence*. The idea here is that isolated quantum systems undergo a certain type of evolution (*unitary* evolution) that classical mechanics explicity violates. Thus there is no hope of obtaining complete correspondence between isolated quantum and

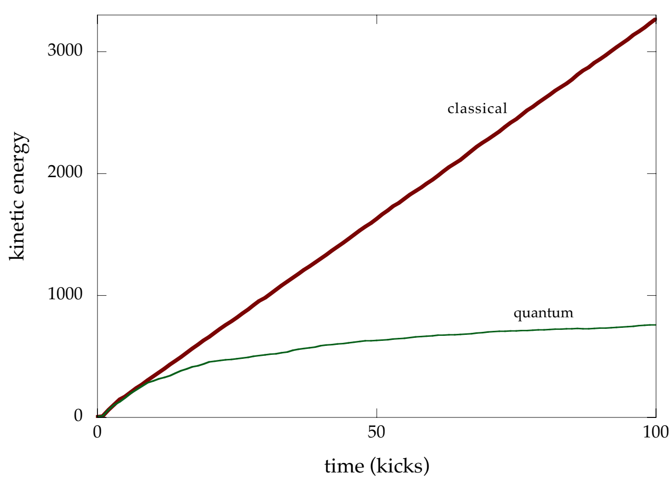

Figure 1: Average kinetic energy for classical and quantum kicked-rotor evolution (simulation). The classical and quantum evolutions only agree for short times. For later times, the classical evolution continues to diffuse, while diffusion is suppressed in the quantum evolution.

classical systems. However, it is important to take into account the fact that most everyday macroscopic systems are not isolated from their environments (being in thermal contact with surrounding light, atmosphere, and other objects). When one does this, the quantum-mechanical evolution is no longer unitary, and there is hope of obtaining correspondence. The most important effect of the surrounding environment is essentially a noisy perturbation on the system of interest. This environmental noise destroys quantum interferences. Since dynamical localization relies on interferences, one could expect such a noisy perturbation to restore classical behavior in the quantum kicked rotor.

Experimentally, we can study the quantum–classical transition in the kicked rotor by starting with the atoms well isolated to observe localization, then adding some form of noise in a controlled way to break localization. In doing these experiments, one must be careful: the signature of destroyed localization is increased momentum diffusion, but of course adding noise will cause diffusion on its own. To first demonstrate delocalization, we added a noisy perturbation in the form of spontaneous emission. The light that caused the spontaneous emission was the same light that we used to cool and trap the atoms while preparing them for the experiment, so this perturbation tends to reduce diffusion. However, as we increased the light level we saw increased diffusion (heating), indicating

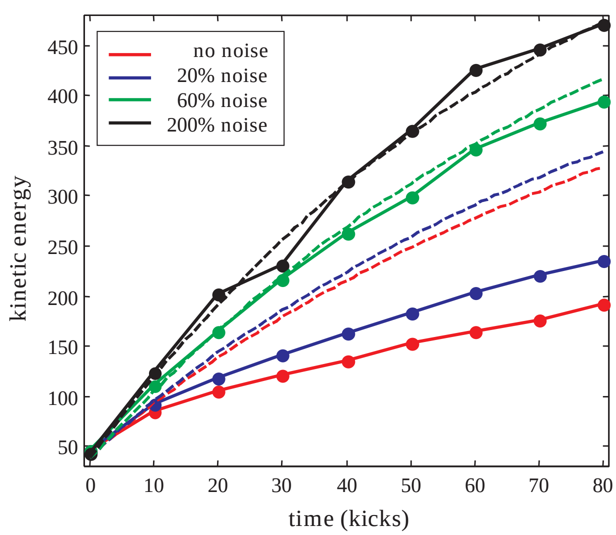

Figure 2: Average kinetic energy for (quantum) experimental measurement (points and solid lines) compared to a classical model of the experiment (dashed lines) for 4 different levels of amplitude noise. As the noise level increases the agreement between quantum and classical improves.

that localization was indeed being destroyed.

To more fully examine the quantum–classical transition, it was necessary to perform a more quantitative experiment. To do this we added noise in the form of controlled fluctuations in the optical lattice intensity. We then developed a detailed classical model of the kicked rotor coupled to intensity noise, which accounted for a variety of systematic effects in the experimental measurements that affected the momentum measurements. The results are summarized in Fig. 2, which compares the experimental (quantum) measurements of the kinetic energy with the classical model for various levels of applied noise. For low noise levels, localization causes large discrepancies in the late-time energies. On the other hand, the agreement is very good for large noise levels, indicating a return to classical behavior.

Chaos-Assisted Tunneling

The other effect studied in this dissertation is *tunneling* in phase space. The experimental setup for studying this effect is very similar to the setup for the kicked-rotor experiment, except that the optical-lattice intensity is modulated sinusoidally in time [leading to a potential of the form $\alpha \cos^2(t) \cos(x)$], giving rise to a realization of the *amplitude-modulated pendulum*. The kicked-rotor system described above is strongly chaotic in the sense that nearly all possible trajectories are chaotic; this system, on the other hand, is *mixed* in that both chaotic and stable trajectories are prevalent in phase space. To visualize this situation, consider the plot of the classical phase space in

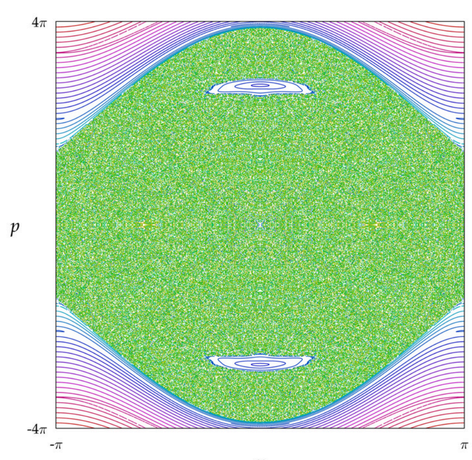

Figure 3: Plot of the classical phase space for typical experimental parameters.

Fig. 3 for the amplitude-modulated pendulum. This plot is constructed by plotting the trajectory coordinates once per modulation period, with different colors corresponding to different initial conditions. The chaotic trajectories appear in the diffuse, noisy region, while the stable trajectories form well-defined contours in phase space.

The phase-space features relevant to the experiments here are the two smaller stable areas within the larger chaotic region; these are picturesquely referred to as "islands of stability in a sea of chaos." The important feature of these islands is that a classical trajectory within an island cannot escape it and visit the other island. Quantum mechanically, though, classically forbidden transport is often allowed, and such transport is referred to as quantum tunneling. Commonly, tunneling is discussed in the context of transport through a potential energy barrier; in barrier tunneling, a classical trajectory has insufficient energy to cross the barrier, but a corresponding quantum wave packet nevertheless has some probability of crossing the barrier. Because the optical potential in the experiments here is modulated, there is no potential barrier in the usual sense. The classical transport between the islands of stability is forbidden by the system dynamics, and thus the quantum tunneling between the islands is referred to as *dynamical tunneling*.

We can understand dynamical tunneling in this system in terms of a simple two-state model. Assume that there are two states, $|\mathrm{U}\rangle$ and $|\mathrm{L}\rangle$, which are localized on the upper and lower islands, respectively. Owing to the

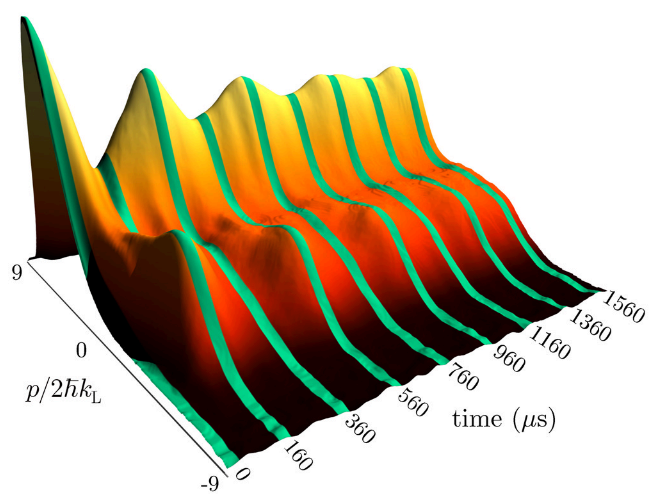

Figure 4: Experimentally measured evolution of the atomic momentum-distribution, showing coherent tunneling oscillations. The distribution here is sampled every 40 μs (every 2 modulation periods).

reflection symmetry of the phase space about $p = 0$, the relevant Floquet states (the generalization of energy eigenstates in periodically driven systems) can be represented as even and odd superpositions of the localized states, $|\pm\rangle = (|\mathrm{U}\rangle \pm |\mathrm{L}\rangle)/\sqrt{2}$. A wave packet initially localized on one of the stability islands thus excites a coherent superposition of these Floquet states. As the Floquet states dephase, the wave packet tunnels periodically between the two islands.

We observe dynamical tunneling of the atoms in the experiment, as shown in Fig. 4. This figure shows the measured atomic momentum distribution as a function of time. Initially, the atoms are concentrated at a momentum corresponding to one of the stability islands. Several coherent tunneling oscillations to the symmetric momentum state and back are apparent in this measurement.

The primary challenge in observing dynamical tunneling experimentally lies in the preparation of the initial state. In brief, this state-preparation process involves trapping and cooling the atoms in a standard magneto-optic trap, applying laser light to polarize the atomic spins into a state that is insensitive to magnetic fields, driving a two-photon transition to select out the coldest 0.3% of the atoms, and then manipulating the atoms with the optical lattice to produce a nearly minimum-uncertainty wave packet localized on one of the stability islands.

The two-state model only focuses on the role of the stability islands in the tunneling. However, the chaotic region in phase space also strongly influences the tunneling. The basic mechanism for this is the fact that a state in the chaotic region can become coupled to the doublet, causing the tunneling rate to increase by as much as several orders of magnitude. This effect is known as *chaos-assisted tunneling*.

We have demonstrated that the tunneling seen in the experiment is in fact chaos-assisted tunneling by observing several distinctive signatures. First, it is possible to compare the tunneling in the amplitude-modulated pendulum to a similar dynamical-tunneling process (Bragg scattering) in the ordinary pendulum, where chaos is absent; we observe that the tunneling rate in the modulated system is orders of magnitude faster than what is expected for Bragg scattering. Furthermore, we observe multiple, simultaneous tunneling oscillations with different rates, which indicate that more than two states are important in understanding the tunneling transport. This is reinforced by the observation that one of the components of the oscillations involves the transfer of population into the chaotic region in phase space. Finally, as we increase the intensity of the optical lattice, we observe that the tunneling rate *decreases*, which is counter to the situation expected for ordinary dynamical tunneling.

Like dynamical localization, chaos-assisted tunneling is a coherent quantum effect and should be susceptible to noise. We have again applied noise to the amplitude-modulated pendulum in the form of intensity fluctuations in the optical lattice. Large levels of applied noise destroy the tunneling oscillations, thus restoring classical behavior. More interestingly, we are able to change the experimental parameters in such a way that the atomic system has a larger action scale (compared to Planck's constant) but without changing the classical behavior of the system. This effectively moves the experimental system toward the classical limit while fixing the underlying classical dynamics. When we do so, we find that the tunneling is more sensitive to noise for the "more classical" set of parameters.