Chaos-Assisted Tunneling in Atom Optics

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Daniel A. Steck

Collaborators:
Windell H. Oskay
Mark G. Raizen

Atom Optics Laboratory
Department of Physics
The University of Texas at Austin

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Barrier Tunneling

• Phase-space tunneling is related to tunneling in the double well potential

• Uncoupled limit: degenerate energy doublets

• Coupling (via the finite barrier) between the two wells leads to broken degeneracy and doublet structure

• Tunneling (Rabi oscillations) occur as the doublet states dephase

• Symmetry is important for tunneling, causes degeneracy in uncoupled limit (resonant Rabi oscillations)
Phase Space

• Graphical representation of the equations of motion

• Pendulum phase space:

• Integrable systems: trajectories confined to surfaces of lower dimension in phase space
Double-Well Phase Space

• Can think of barrier tunneling in phase space

![Phase Space Diagram]

• Classical transport is forbidden: trajectories cannot cross invariant surfaces

• Quantum transport allowed: quantum paths can cross invariant surfaces, but are exponentially suppressed

• Leads to universal scaling: $\omega \sim \exp(-1/\hbar)$
Optical Lattices and Atom Optics

• Formed by retroreflecting a laser beam -- standing wave
  - stationary, 1-D sinusoidal intensity pattern:

• Far-detuned regime:
  - spontaneous (random) scattering negligible
  - intensity pattern creates spatial potential

• Atomic motion equivalent to pendulum:
Amplitude Modulation

• Full amplitude modulation of standing wave intensity:

\[ H = \frac{p^2}{2} - 2\alpha \cos^2(\pi t) \cos(x) \]

• Can rewrite potential as sum of 3 terms:

\[-\alpha \cos(x) - \frac{\alpha}{2} \cos(x + 2\pi t) - \frac{\alpha}{2} \cos(x - 2\pi t)\]

- one stationary and two moving lattices (pendula)

• Phase space contains three pendulum-like features:

• Want to look for tunneling between two symmetry-related structures
Chaos in Phase Space

- As $\alpha$ (well depth) increases, competition between the three modes of motion leads to chaos:

  \[ \alpha = 0.6 \]

  \[ \alpha = 2.0 \]

  \[ \alpha = 6.0 \]
Dynamical Tunneling

- In our system, islands play the role of the two wells
  - Islands case localization of Floquet states
  - Transport out of islands is classically forbidden

- Simplest picture: tunneling between symmetric islands proceeds as symmetric/antisymmetric Floquet-state pair dephase

- This tunneling is *dynamical tunneling*: transport is forbidden by the dynamics, not a potential barrier

- Predicted by Davis and Heller, 1981

- Also under study by NIST/U. Queensland collaboration
“Simple” Experiment
or “how not to observe tunneling”

• Simplified approach: cool cesium atoms in a 3-D optical lattice to 400 nK (Δp = 1.4ℏk_L)

• Adiabatically turn on 1-D standing wave
  - size is three times that of a minimum-uncertainty packet

• Boost wave packet to match island velocity

• Modulate lattice to realize amplitude modulated pendulum...
Measurement Sequence

1. Magneto-optic trap/preparation (5 s)

2. (State Preparation) (1 ms)

3. Time-Dependent Optical Lattice (1 ms)

4. Free expansion (15 ms)

5. Freezing optical molasses/imaging (1 ms)

• Technique for measuring momentum distributions/energies
Tunneling in Phase Space?

- Experimental momentum distributions vs. time:
Symmetries

- **Classical symmetry**: satisfied due to island structure in phase-space

- **Quantum symmetry**: quantum mechanics imposes an additional symmetry
  - atoms change momentum in multiples of $2\hbar k_L$
  - tunneling requires that states are coupled to their reflections about $p = 0$ via these discrete steps

- Consequence: only certain “integer states” can tunnel

- Analogous to asymmetric double well or broken time-reversal symmetry

- Requires subrecoil velocity selection
Raman Velocity Selection

- Use stimulated, two-photon transition between cesium ground states:

  \[
  F=4, m=0 \quad \text{and} \quad F=3, m=0
  \]

- If the two beams are counterpropagating, the atomic momentum enters into the resonance condition:

  \[
  \omega_2 - \omega_2 = 2\pi \cdot 9.2 \text{ GHz} + \frac{p}{\hbar k_L} \cdot 4\omega_r
  \]

- Velocity selection procedure:
  1. Optically pump to \( F = 4, m = 0 \)
  2. Tag atoms with proper velocity into \( F = 3 \)
  3. Push away \( F = 4 \) atoms with resonant light

- Result: subrecoil atoms near \( p = 0 \)
State Preparation

- Create localized state while preserving subrecoil structure

1. Begin with subrecoil sample from Raman tagging

2. Turn on 1-D standing wave adiabatically
   - atoms become localized in the lattice wells, also heating
   - subrecoil slices within overall profile ⇒ coherence over several wells
   - minimum uncertainty for deep wells

3. Sudden shift of standing-wave phase
   - using phase modulator before standing-wave retroreflector

4. Free evolution of atoms in optical lattice
   - nearly harmonic evolution until \( p \) is maximized
Initial Condition in Phase Space

- Initial conditions with Raman $\Delta p = 0.03 \times 2\hbar k_L$
- Other parameters: $\alpha = 10.5$, $k = 2.08$
Tunneling in Phase Space

- Experimental momentum distributions vs. time, this time with Raman $\Delta p = 0.03 \times 2\hbar k_L$ (800 µs tag):

- Four oscillations before damping away
- Coherent, 16-photon transition
- Parameters: $\alpha = 10.5$, $\bar{k} = 2.08$
Island Dependence

- Verify that tunneling is indeed related to classical island structure, by inserting delay time after state preparation:
Raman Tagging Effects

- Shift locations of velocity slices within overall shape by changing Raman detuning:

- Vary width of Raman velocity selection:

- Incomplete tunneling due mostly to Raman tag width
Chaos-Assisted Tunneling

• Tunneling is “assisted” by the chaos in the sense that tunneling can be greatly enhanced by the presence of chaos

• Enhancement can be understood in two ways:
  - Quantum paths: paths through chaotic region are not attenuated as strongly as those that cross KAM tori
  - Avoided crossings: tunneling doublet can interact with a third chaotic state, prying apart the doublet

• Should be strong fluctuations in the tunneling rate as parameters vary; no universal dependence
Bragg Scattering

• This tunneling is reminiscent of another form of tunneling in optical lattices: Bragg scattering

• Dynamical tunneling in a stationary (integrable) lattice: atom can reverse direction quantum mechanically but not classically

• Two-state process: transition between two symmetric plane-wave states

• Intermediate states are negligibly populated:
Comparison with Integrable Tunneling

• Natural integrable counterpart of tunneling: Bragg scattering

• Consider time-averaged potential dynamics: pendulum

• Classical transport in pendulum forbidden by separatrix

• Bragg scattering provides similar transport mechanism in momentum

• No Bragg oscillations over time of experiment

• 8th order Bragg period for \( \alpha = 10.5, \, \vec{k} = 2.1 \) is 1 s

• Also 32-photon tunneling

• 16th order Bragg period for \( \alpha = 11.2, \, \vec{k} = 1.0 \) is 20 yr
Tunneling Variation

• Study dependence of tunneling on $\alpha$ ($\bar{k} = 2.08$)

• Tunneling only visible in a relatively narrow range of $\alpha$
Tunneling Rate Variation

- Study dependence on tunneling rate vs. $\alpha$ ($\vec{k} = 2.08$)

- Rate shows overall decrease with $\alpha$

- Also observe both one- and two-frequency behavior

- Two-frequency behavior consistent with center of avoided crossing
High Time Resolution

- Sample momentum distribution 10 times/modulation period
- Measurement spans 1 tunneling period, $\alpha = 7.7$, $\bar{k} = 2.08$

- Oscillations on 3 time scales:
  1. longest is tunneling
  2. shortest is classical island motion
  3. intermediate is influence of third level
Continuous Phase Space Evolution

• Islands move continuously between stroboscopic samples

• Islands move together during first half of modulation cycle:
Strongly Coupled Regime

• Measurement for $\alpha = 17, \bar{k} = 2.08$:

  - Fast, irregular oscillations
  - Classical islands have broken down
  - Quantum states can no longer be grouped into doublets
Noise and Decoherence

- Tunneling behavior poses problem for classical limit
  - two-state tunneling: $\exp(-S/\hbar)$ scaling of tunneling rate ensures that macroscopic tunneling doesn’t happen
  - three-state tunneling: no universal scaling of tunneling rate, so need alternate mechanism for classical behavior

- Tunneling is a coherent effect, so can be destroyed by noise or interaction with the environment (decoherence)

- Study experimentally by adding noise to the optical lattice intensity:

$$H = \frac{p^2}{2} + 2\alpha[1 + \varepsilon(t)] \cos(x) \cos^2(\pi t)$$
Amplitude Noise Effects

- Can compare effects for different effective Planck constant $\bar{k}$ ($\alpha = 11.2$)

- Noise level is the standard deviation compared to the average intensity

- Bandwidth-limited to the same scaled cutoff frequency for meaningful comparison

- Smaller $\bar{k}$ is more sensitive to the noise
Summary

- Studied chaos-assisted tunneling of cesium atoms in a modulated standing wave
- Studied several features of dynamical tunneling:
  - sensitivity to classical phase-space structure
  - sensitivity to momentum class
- Studied several features specific to CAT
  - enhancement relative to integrable tunneling
  - extra oscillation in tunneling process
  - avoided crossing behavior by varying well depth
- Noise effects
  - damping of oscillations, relaxation
  - different sensitivity for different scaled Planck constant