Appendix C

Phase Space Gallery II: Amplitude-Modulated Pendulum

Now we examine the phase space for the other system that we study in this dissertation, the amplitude-modulated pendulum. This system is described by the Hamiltonian

\[ H(x, p, t) = \frac{p^2}{2} - 2\alpha \cos^2(\pi t) \cos(x), \]  

which is just the pendulum Hamiltonian with a single-frequency modulation of the well depth. As the time dependence here can be decomposed into three frequencies, the phase-space is dominated by the three corresponding primary resonances, located at \( p = 0 \) and \( p = \pm 2\pi \). These three resonances form as \( \alpha \) increases from zero, and they dissolve into the surrounding chaotic sea as \( \alpha \) continues to grow. Especially dramatic is the “molting” behavior of the islands, where they grow an island chain and then shed it into the chaotic sea; this can be seen, for example, for a period-4 chain in the center island around \( \alpha = 3.2 \), and a period-4 island chain in the outer islands around \( \alpha = 5.2 \). Also interesting is that the remnants of the center island disappear around \( \alpha = 11 \), but the island makes a strong reappearance around \( \alpha = 18 \).

In the following graphics, the trajectory coordinates are plotted, sampled at unit times \( t = n \) (for integer \( n \)). The phase plots here show about 60 different trajectories, with around 4000 iterations per trajectory. To retain the symmetry of the phase space, the three symmetric images \((x, -p), (-x, p), \) and \((-x, -p)\) of each \((x, p)\) point is also plotted.

The phase-space plots in this gallery were again hand-coded directly in POSTSCRIPT, where the code contained two embedded integrators, a fixed-step, second-order Stoermer routine and a fourth-order Runge-Kutta routine. Despite the lower order of the Stoermer method, it was much more accurate for the same step size than the Runge-Kutta integrator. These graphics files were again rasterized before inclusion in the PDF-formatted version of this document, as they require extensive processing time compared to the standard-map phase plots.
Appendix C. Phase Space Gallery II: Amplitude-Modulated Pendulum

$\alpha = 0.0$

$\alpha = 0.2$

$\alpha = 0.4$

$\alpha = 0.6$

$\alpha = 0.8$

$\alpha = 1.0$
Appendix C. Phase Space Gallery II: Amplitude-Modulated Pendulum

\[ \alpha = 2.4 \]

\[ \alpha = 2.6 \]

\[ \alpha = 2.8 \]

\[ \alpha = 3.0 \]

\[ \alpha = 3.2 \]

\[ \alpha = 3.4 \]
\[ \alpha = 3.6 \]

\[ \alpha = 3.8 \]

\[ \alpha = 4.0 \]

\[ \alpha = 4.2 \]

\[ \alpha = 4.4 \]

\[ \alpha = 4.6 \]
Appendix C. Phase Space Gallery II: Amplitude-Modulated Pendulum

\[ \alpha = 4.8 \]

\[ \alpha = 5.0 \]

\[ \alpha = 5.2 \]

\[ \alpha = 5.4 \]

\[ \alpha = 5.6 \]

\[ \alpha = 5.8 \]
Appendix C. Phase Space Gallery II: Amplitude-Modulated Pendulum

\[ \alpha = 7.2 \]

\[ \alpha = 7.4 \]

\[ \alpha = 7.6 \]

\[ \alpha = 7.8 \]

\[ \alpha = 8.0 \]

\[ \alpha = 8.2 \]
Appendix C. Phase Space Gallery II: Amplitude-Modulated Pendulum

\[ \alpha = 9.6 \]

\[ \alpha = 9.8 \]

\[ \alpha = 10.0 \]

\[ \alpha = 10.4 \]

\[ \alpha = 10.8 \]

\[ \alpha = 11.2 \]