Noise Analysis: Libbrecht–Hall Circuit

Note: all noise estimates here are in $\mathrm{pA}/\sqrt{\mathrm{Hz}}$, which should be squared when compared to the spectrum-amplifier measurements.

1 Main-Loop Noise Estimate

The noise in the regulated current is set mainly by the LT1028 regulating op-amp and the $50\,\Omega$ sense resistor. The LT1028 specifies a voltage noise density of $0.85\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ for the better LT1028AM/AC variant at 1 kHz. This is input-referenced, and is translated to a current noise density by the $50\,\Omega$ sense resistor to $17\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$. The sense resistor itself contributes Johnson noise with rms voltage noise density

$$\delta v_{\mathrm{jn}} = \sqrt{4k_{\mathrm{B}}TR} = (0.128\,\mathrm{nV}/\sqrt{\mathrm{Hz}})\sqrt{R} \tag{1}$$

at 25°C, with the resistance R in ohms. This gives a voltage noise density of $0.91\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$, corresponding to a current noise density of $18\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$. Adding the two noises in quadrature gives $25\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, in agreement with Libbrecht and Hall.

Note that the $1\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ input noise current density should also be added (in quadrature) to this result, because current flowing through the sense resistor also can flow into the inverting input of the amplifier. However, this is a negligible correction.

Notice that the setup here is roughly optimal for the selected amplifier, since the amplifier and Johnson noises are about the same. A larger sense resistor decreases the effect of the amplifier noise, but increases the Johnson noise. A smaller sense resistor has the opposite effect. For example, a $30\,\Omega$ sense resistor increases the effective amplifier noise to $28\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, while decreasing the Johnson noise to $0.70\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$, which is equivalent to $23\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$. The total is then $36\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$, which is worse off, at least using the 1 kHz noise figure. A smaller resistor may better optimize for larger noise figures at lower frequencies (for example, $1.0\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ for the LT1028 at 10 Hz).

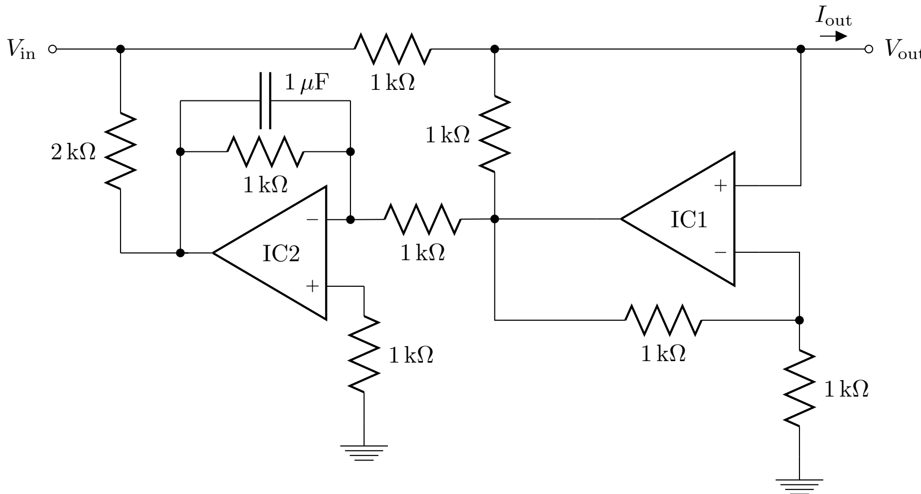
1.1 Why Diodes?

The diodes on the power-supply inputs don't help with noise suppression, but rather are there for protection of the op-amp inputs. For ultralow-noise amplifiers like the LT1028 or AD797, the inputs don't have current-limiting input resistors, but have back-to-back protection diodes. The diodes help to protect the inputs in situations where the amplifier is unpowered (or unusually powered, as in turn-on or turn-off transients). This is explained in more detail by Ardizzoni[1]

2 Input-Stage Noise

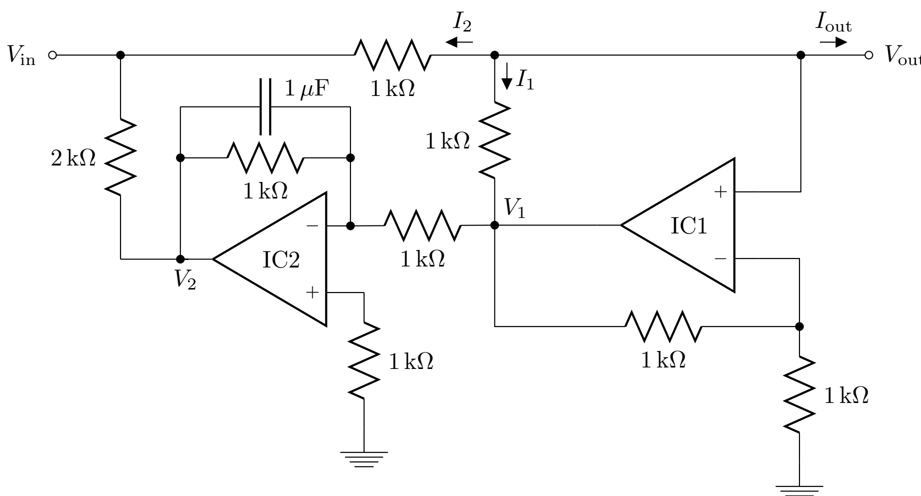
The modulation input stage of the Libbrecht–Hall circuit is shown below. The voltage V_{out} at the output is held to a fixed voltage, determined by the output current and the load (diode laser) effective resistance.

[1] John Ardizzoni, "Protecting Off-Amps" `http://www.analog.com/library/analogdialogue/archives/42-10/off_amps.html`



2.1 Operation

First, IC1 causes $I_{\rm out} = V_{\rm in}/(1\,{\rm k}\Omega)$, and is independent of $V_{\rm out}$. to see this, we first draw in some key voltages and currents.



Then IC1 acts as a noninverting amplifier of gain 2,

$$V_1 = 2V_{\rm out}. \tag{2}$$

This sets I_1 to

$$I_1 = \frac{V_{\rm out} - V_1}{1\,{\rm k}\Omega} = \frac{V_{\rm out} - 2V_{\rm out}}{1\,{\rm k}\Omega} = -\frac{V_{\rm out}}{1\,{\rm k}\Omega}, \tag{3}$$

and I_2 is set by

$$I_2 = \frac{V_{\rm out} - V_{\rm in}}{1\,{\rm k}\Omega}. \tag{4}$$

Then the output current is

$$I_{\rm out} = -I_1 - I_2 = \frac{V_{\rm in}}{1\,{\rm k}\Omega}, \tag{5}$$

since V_{out} cancels. Note that IC2 did not contribute to the output, since its output voltage is "overridden" by V_{in}.

Next, the function of IC2 is to handle the case where V_{in} is disconnected (i.e., not held at any particular voltage), so that $I_{\text{out}} = 0$, independent of V_{out}. In this case, V_{in} floats, and IC2 acts as a unity gain inverter, so we also have

$$V_2 = -V_1 = -2V_{\text{out}}. \tag{6}$$

Now I_2 is set by this voltage,

$$I_2 = \frac{V_{\text{out}} - V_2}{3\,\text{k}\Omega} = \frac{V_{\text{out}} + 2V_{\text{out}}}{3\,\text{k}\Omega} = \frac{V_{\text{out}}}{1\,\text{k}\Omega}, \tag{7}$$

and thus the output current is

$$I_{\text{out}} = -I_1 - I_2 = 0, \tag{8}$$

independent of V_{out}.

2.2 Noise Estimate

First, consider the noise contributed by the IC1 portion of the circuit. The $1\,\text{k}\Omega$ sense resistor contributes $4.1\,\text{nV}/\sqrt{\text{Hz}}$ of Johnson noise, while the two resistors connected to the inverting input act as a single resistor of (parallel) resistance $500\,\Omega$, contributing another $2.9\,\text{nV}/\sqrt{\text{Hz}}$ of noise. The amplifier itself is specified as an OP27, with $3.0\,\text{nV}/\sqrt{\text{Hz}}$ at 1 kHz for the better (OP27A/E) grade, and an input noise current density of $0.4\,\text{pA}/\sqrt{\text{Hz}}$. Summed in quadrature, the voltage noises total $5.8\,\text{nV}/\sqrt{\text{Hz}}$. Converted to a current noise via the $1\,\text{k}\Omega$ sense resistor, this is equivalent to $5.8\,\text{pA}/\sqrt{\text{Hz}}$. The op-amp's input current noise should also contribute, but by comparison it is negligible. Notice that adding this current density to the $25\,\text{pA}/\sqrt{\text{Hz}}$ noise from the main regulating amplifier only increases the noise by about 2%, to $26\,\text{pA}/\sqrt{\text{Hz}}$.

That is the noise estimate assuming that the input is held fixed by a noiseless source. With a disconnected input voltage, the input resistors to IC2 contribute the same total $5.8\,\text{nV}/\sqrt{\text{Hz}}$, including the OP27 noise, at the IC2 output. (This is something of an overestimate, as some of the noise is bypassed at higher frequencies by the feedback capacitor.) We should then add the voltage noise due to the $2\,\text{k}\Omega$ resistor ($5.7\,\text{nV}/\sqrt{\text{Hz}}$), for a total of $8.1\,\text{nV}/\sqrt{\text{Hz}}$ at the V_{in} point. This voltage noise is converted to current noise by the $1\,\text{k}\Omega$ resistor, leading to a noise current I_2 in the diagram above of $8.1\,\text{pA}/\sqrt{\text{Hz}}$. Adding this to the I_1 noise that we just calculated, the total current noise is now $10.0\,\text{pA}/\sqrt{\text{Hz}}$. Adding this current density to the $25\,\text{pA}/\sqrt{\text{Hz}}$ noise from the main regulating amplifier increases the noise to $27\,\text{pA}/\sqrt{\text{Hz}}$. So when running this circuit, shorting the input to ground when not in use should reduce the noise in the circuit slightly.

The Durfee group uses very-low-noise op amps here. Recalculating for the noise figure of the LT1028 gives noises of $5.1\,\text{nV}/\sqrt{\text{Hz}}$ for each amplifier separately (vs. $5.8\,\text{nV}/\sqrt{\text{Hz}}$), leading to a total input noise of $9.2\,\text{pA}/\sqrt{\text{Hz}}$ (vs. $10.0\,\text{pA}/\sqrt{\text{Hz}}$), a modest reduction in noise, especially when added to the main $25\,\text{pA}/\sqrt{\text{Hz}}$ noise (the total noise is slighly lower, but is still $27\,\text{pA}/\sqrt{\text{Hz}}$ at this level of accuracy).

3 MIT/Northwestern Variant

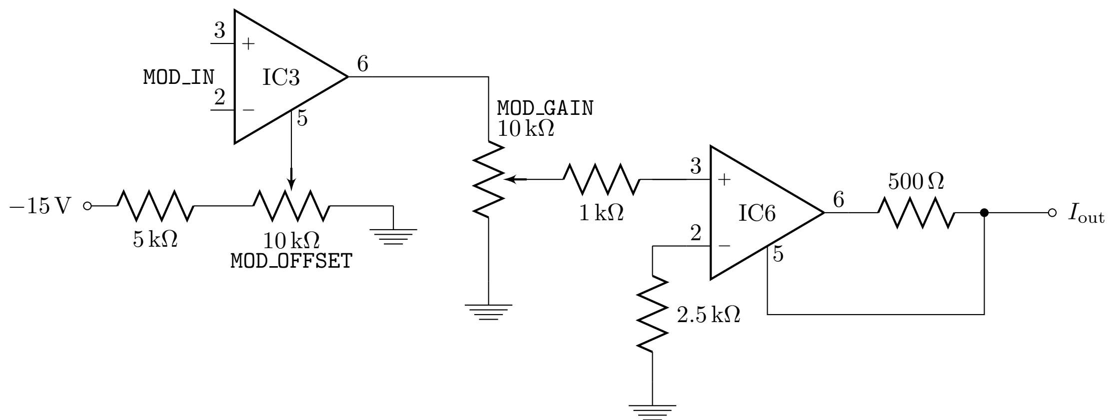
3.1 Main Current Regulation

This circuit uses an AD711 in place of the LT1028 in the nominal design. At 1 kHz the input noise voltage density is specified at $18\,\text{nV}/\sqrt{\text{Hz}}$. (The input noise current is completely ignorable at $0.01\,\text{pA}/\sqrt{\text{Hz}}$.) This corresponds to a current noise density of $360\,\text{pA}/\sqrt{\text{Hz}}$, which completely dominates the sense-resistor Johnson noise, and is much larger than in the original circuit.

It thus seems best to revert to a lower-noise op amp. The LT1028 has a noise peak, which may be mitigated by picking another amplifier with similarly low noise. Good candidates are: AD797, LT1115, LT1128 (basically a compensated LT1028), ADA4898-1.

3.2 Modulation Input

The representative parts of the modulation-input section are shown below.



Both IC's here are INA114 instrumentation amplifiers. Neither has a gain resistor connected, so they both operate as unity-gain amplifiers. These are fairly complex circuits, and the noise specifications are sparse, so this will be a rough estimate. The data sheet specifies a noise voltage of $11\,\text{nV}/\sqrt{\text{Hz}}$ at 1 kHz and a gain of 1000. This corresponds to a gain resistor of $50\,\Omega$, which should contribute negligibly to the noise compared to the internal $25\,\text{k}\Omega$ resistors, so this seems like a reasonable noise figure to use for unity gain (note that the noise figure is input-referenced, and thus gain-independent). However, see the discussion below about low-noise instrumentation amps—the noise in this amplifier is probably *much* worse, by a factor of 5 or so in noise voltage density.

The IC6 circuit is a simple instrumentation-amp current source, using a $500\,\Omega$ sense resistor, giving an output $(V_+ - V_-)/500\,\Omega$ in terms of the input voltages. Thus, the voltage noise corresponds to $22\,\text{pA}/\sqrt{\text{Hz}}$, which is already about the same as the main-regulator noise in the original circuit. The sense resistor contributes an additional $6\,\text{pA}/\sqrt{\text{Hz}}$. The negative-input resistor (which seems to serve no purpose, as it doesn't balance any impedances, and the INA114 inputs should be very high impedance anyway) contributes another $13\,\text{pA}/\sqrt{\text{Hz}}$. The $10\,\text{k}\Omega$ `MOD_GAIN` pot and $1\,\text{k}\Omega$ input resistor (again, the latter serves no purpose) are in the worst case equivalent to a $6\,\text{k}\Omega$ resistor (at the halfway position), contributing another $20\,\text{pA}/\sqrt{\text{Hz}}$ of noise. Adding these together (in quadrature) totals $33\,\text{pA}/\sqrt{\text{Hz}}$, substantially greater than the main-circuit noise of $25\,\text{pA}/\sqrt{\text{Hz}}$ in the original circuit, which would total $41\,\text{pA}/\sqrt{\text{Hz}}$ if added together.

We can sum up the (voltage) noise of the IC3 amplifier as follows: The amplifier itself contributes $11\,\text{nV}/\sqrt{\text{Hz}}$. The reference input is connected to the `MOD_OFFSET` pot, which in the worst case acts as a $7.5\,\text{k}\Omega$ resistor, for another $11\,\text{nV}/\sqrt{\text{Hz}}$ noise. Each input also has a $100\,\text{k}\Omega$ resistor to ground (not shown); with an unconnected `MOD_IN`, each input also contributes $40\,\text{nV}/\sqrt{\text{Hz}}$ of noise ($57\,\text{nV}/\sqrt{\text{Hz}}$ total for both inputs). Totaling these all, the voltage noise is $59\,\text{nV}/\sqrt{\text{Hz}}$, dominated by the input resistors. This corresponds to $118\,\text{pA}/\sqrt{\text{Hz}}$ of current noise, in the worst case where `MOD_GAIN` is maximum. This would dominate the noise in the entire circuit.

Recommendations to improve this: First, dump the input resistors to IC3; the negative input (pin 2, presumably connected to ground from the input source) should be connected to ground with a $10\,\Omega$ resistor and $0.01\,\mu$F capacitor in parallel. Pin 3 should have no connection to ground; this input can be shorted in case it is unused. Second, the `MOD_OFFSET` should be derived from a stable, lower-voltage reference (e.g., a buffered LM399), eliminating the need for a voltage-limit resistor, and then the pot should be reduced to 1 or $2\,\mathrm{k}\Omega$. Third, `MOD_GAIN` should be a 1 or $2\,\mathrm{k}\Omega$ pot. Fourth, bypass both input resistors to IC6. Fifth, increase the sense resistor to $1\,\mathrm{k}\Omega$. Last, lower-noise amplifiers should be used. This is somewhat tricky, as instrumentation amplifiers are inherently a bad choice in this application. For example, the AD8429[2] is a low-noise amplifier, specifying $1\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ input noise, but $45\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ *output* noise, which dominates for $G = 1$ [the total input-referenced noise is basically the input noise added in quadrature with (output noise)/G]. Thus, the calculations above could be wildly off, because the noise is specified at $G = 1000$, when the output-stage noise is negligible. See also the LT1167 data sheet,[3] with $7.5\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ input noise but $67\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ output noise. This is a better comparison to the INA114 (or INA128).

Suppose we make all these changes, assuming $1\,\mathrm{k}\Omega$ pots and AD8429 instrumentation amplifiers. Then IC6 has $45\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ intrinsic noise, with $4.0\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ for the sense resistor and $2.8\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$ for the gain pot. This totals to $45\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$, or $45\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$ noise current. Already this is still worse than the original circuit. (Note that this is greater than the original calculation, but we almost certainly underestimated the noise of the INA114.) Then IC3 contributes similar noise, for a total of $64\,\mathrm{pA}/\sqrt{\mathrm{Hz}}$. This performance is much worse than the original circuit, and this should be replaced by a much quieter design (based on op amps) or the original.

[2] `http://www.analog.com/static/imported-files/data_sheets/AD8429.pdf`
[3] `http://cds.linear.com/docs/en/datasheet/1167fc.pdf`